4. When the powers have the same base, use the exponent law for products and add the exponents.

a) $5^5 \times 5^4 = 5^{(5+4)}$
$= 5^9$

b) $10^2 \times 10^{11} = 10^{(2+11)}$
$= 10^{13}$

c) $(-3)^3 \times (-3)^3 = (-3)^{(3+3)}$
$= (-3)^6$

d) $21^6 \times 21^4 = 21^{(6+4)}$
$= 21^{10}$

e) $(-4)^1 \times (-4)^3 = (-4)^{(1+3)}$
$= (-4)^4$

f) $6^{12} \times 6^3 = 6^{(12+3)}$
$= 6^{15}$

g) $2^0 \times 2^4 = 2^{(0+4)}$
$= 2^4$

h) $(-7)^3 \times (-7)^0 = (-7)^{(3+0)}$
$= (-7)^3$

5. When the powers have the same base, use the exponent law for quotients and subtract the exponents.

a) $4^5 \div 4^3 = 4^{(5-3)}$
$= 4^2$

b) $8^9 \div 8^6 = 8^{(9-6)}$
$= 8^3$

c) $15^{10} \div 15^0 = 15^{(10-0)}$
$= 15^{10}$

d) $(-6)^8 \div (-6)^3 = (-6)^{(8-3)}$
$= (-6)^5$

e) $\frac{2^{12}}{2^{10}} = 2^{(12-10)}$
$= 2^2$

f) $\frac{(-10)^{12}}{(-10)^6} = (-10)^{(12-6)}$
$= (-10)^6$

g) $\frac{6^5}{6^1} = 6^{(5-1)}$
$= 6^4$

h) $\frac{(-1)^5}{(-1)^4} = (-1)^{(5-4)}$
$= (-1)^1$

10. a) $10^2 \times 10^2 + 10^4 = 10^{(2+2)} + 10^4$
$= 10^4 + 10^4$
$= 10\,000 + 10\,000$
$= 20\,000$

b) $10^3 \times 10^3 - 10^3 = 10^{(3+3)} - 10^3$
$= 10^6 - 10^3$
$= 1\,000\,000 - 1000$
$= 999\,000$

c) $10^{11} - 10^3 \times 10^6 = 10^{11} - 10^{(3+6)}$
$= 10^{11} - 10^9$
$= 100\,000\,000\,000 - 1\,000\,000\,000$
$= 99\,000\,000\,000$

d) $10^1 + 10^5 \times 10^2 = 10^1 + 10^{(5+2)}$
$= 10^1 + 10^7$
$= 10 + 10\,000\,000$
$= 10\,000\,010$

e) $10^6 \div 10^2 \times 10^2 = 10^{(6-2)} \times 10^2$
$= 10^4 \times 10^2$
$= 10^{(4+2)}$
$= 10^6$
$= 1\,000\,000$

f) $10^9 \div 10^9 = 10^{(9-9)}$
$= 10^0$
$= 1$

g) $\frac{10^{12}}{10^6} = 10^{(12-6)}$
$= 10^6$
$= 1\,000\,000$

h) $\frac{10^4 \times 10^3}{10^2} = \frac{10^{(4+3)}}{10^2}$
$= \frac{10^7}{10^2}$
$= 10^{(7-2)}$
$= 10^5$
$= 100\,000$

i) $\frac{10^{11}}{10^4 \times 10^2} = \frac{10^{11}}{10^{(4+2)}}$
$= \frac{10^{11}}{10^6}$
$= 10^{(11-6)}$
$= 10^5$
$= 100\,000$

j) $\frac{10^5}{10^3} + 10^2 = 10^{(5-3)} + 10^2$
$= 10^2 + 10^2$
$= 100 + 100$
$= 200$

12. a) In square metres, the area of the field is
$10^4 \times 10^3 = 10^{(4+3)}$
$= 10^7$
$= 10\ 000\ 000$

b) In metres, the perimeter of the field is
$2(10^4 + 10^3) = 2(10\ 000 + 1000)$
$= 2(11\ 000)$
$= 22\ 000$

c) i) All possible dimensions are all the pairs of powers of 10 that have a product of 10^7, so find all possible exponents that have a sum of 7.

10^7 m by 10^0 m
since $10^7 \times 10^0 = 10^{(7+0)}$
$= 10^7$

10^6 m by 10^1 m
since $10^6 \times 10^1 = 10^{(6+1)}$
$= 10^7$

10^5 m by 10^2 m
since $10^5 \times 10^2 = 10^{(5+2)}$
$= 10^7$

10^4 m by 10^3 m
since $10^4 \times 10^3 = 10^{(4+3)}$
$= 10^7$

ii) $2(10^7\text{ m} + 10^0\text{ m}) = 2(10\ 000\ 000\text{ m} + 1\text{ m})$
$= 2(10\ 000\ 001\text{ m})$
$= 20\ 000\ 002\text{ m}$

$2(10^6\text{ m} + 10^1\text{ m}) = 2(1\ 000\ 000\text{ m} + 10\text{ m})$
$= 2(1\ 000\ 010\text{ m})$
$= 2\ 000\ 020\text{ m}$

$2(10^5\text{ m} + 10^2\text{ m}) = 2(100\ 000\text{ m} + 100\text{ m})$
$= 2(100\ 100\text{ m})$
$= 200\ 200\text{ m}$

$2(10^4\text{ m} + 10^3\text{ m}) = 2(10\ 000\text{ m} + 1000\text{ m})$
$= 2(11\ 000\text{ m})$
$= 22\ 000\text{ m}$

d) The exponent laws are helpful for solving area problems because they are used to simplify expressions that include products, and we multiply to determine area. To determine perimeter, we add and there are no exponent laws for determining the sums of powers

13. a) $2^3 \times 2^2 - 2^5 \times 2 = 2^{(3+2)} - 2^{(5+1)}$
$= 2^5 - 2^6$
$= 32 - 64$
$= -32$

b) $3^2 \times 3 + 2^2 \times 2^4 = 3^{(2+1)} + 2^{(2+4)}$
$= 3^3 + 2^6$
$= 27 + 64$
$= 91$

c) $4^2 - 3^0 \times 3 + 2^3 = 4^2 - 3^{(0+1)} + 2^3$
$= 4^2 - 3^1 + 2^3$
$= 16 - 3 + 8$
$= 21$

d) $(-3)^6 \div (-3)^5 - (-3)^5 \div (-3)^3 = (-3)^{(6-5)} - (-3)^{(5-3)}$
$= (-3)^1 - (-3)^2$
$= -3 - 9$
$= -12$

e) $(-2)^4[(-2)^5 \div (-2)^3] + (-2)^4 = (-2)^4 \times (-2)^{(5-3)} + (-2)^4$
$= (-2)^4 \times (-2)^2 + (-2)^4$
$= (-2)^{(4+2)} + (-2)^4$
$= (-2)^6 + (-2)^4$
$= 64 + 16$
$= 80$

f) $-2^4(2^6 \div 2^2) - 2^4 = -2^4 \times 2^{(6-2)} - 2^4$
$= -2^4 \times 2^4 - 2^4$
$= -2^{(4+4)} - 2^4$
$= -2^8 - 2^4$
$= -256 - 16$
$= -272$

g) $(-5)^3 \div (-5)^2 \times (-5)^0 + (-5)^2 \div (-5) = (-5)^{(3-2)} \times (-5)^0 + (-5)^{(2-1)}$
$= (-5)^1 \times (-5)^0 + (-5)^1$
$= (-5)^{(1+0)} + (-5)^1$
$= (-5)^1 + (-5)^1$
$= -5 + (-5)$
$= -10$

15. a) The student multiplied the exponents instead of adding them.
Correct answer:
$4^3 \times 4^4 = 4^{(3+4)}$
$= 4^7$

b) The student divided the exponents instead of subtracting them.
Note: the exponent is inside the brackets, so the negative sign is not part of the base of the power.
Correct answer:
$$\frac{\left(-7^6\right)}{\left(-7^3\right)} = 7^{(6-3)}$$

$= 7^3$

c) The student used the exponent laws even though the bases are different.
Correct answer:
$3^2 \times 2^3 = 9 \times 8$
$= 72$

d) The student multiplied the exponents in the divisor instead of adding them.
Correct answer:
$$\frac{5^8}{5^4 \times 5^2} = \frac{5^8}{5^{(4+2)}}$$
$$= \frac{5^8}{5^6}$$
$= 5^2$
$= 25$

e) The student added all the exponents even though only 2 of them were parts of products of powers.
Correct answer:
$1^2 + 1^3 \times 1^2 = 1^2 + 1^{(3+2)}$
$= 1^2 + 1^5$
$= 1 + 1$
$= 2$

17. a) i) $5^2 + 5^3 = 25 + 125$
$= 150$
ii) $5^2 \times 5^3 = 5^{(2+3)}$
$= 5^5$
$= 3125$

b) I could not use an exponent law to simplify the expression in part i because it is the sum of two powers. I could use an exponent law to simplify the expression in part ii because it is the product of two powers.

18. a) i) $4^3 - 4^2 = 64 - 16$
$= 48$
ii) $4^3 \div 4^2 = 4^{(3-2)}$
$= 4^1$
$= 4$

b) I could not use an exponent law to simplify the expression in part i because it is the difference of two powers. I could use an exponent law to simplify the expression in part ii because it is the quotient of two powers

20. I found 8 pairs of powers with the same base that have a product of 64:
$8^1 \times 8^1 = 64$
$8^0 \times 8^2 = 64$
$4^3 \times 4^0 = 64$
$4^2 \times 4^1 = 64$
$2^6 \times 2^0 = 64$
$2^5 \times 2^1 = 64$
$2^4 \times 2^2 = 64$
$2^3 \times 2^3 = 64$

I found 8 pairs of powers with different bases that have a product of 64:

$8^1 \times 2^3 = 64$
$8^0 \times 4^3 = 64$
$8^0 \times 2^6 = 64$
$2^6 \times 4^0 = 64$
$4^3 \times 2^0 = 64$
$8^2 \times 2^0 = 64$
$4^2 \times 2^2 = 64$
$2^4 \times 4^1 = 64$

There are many more pairs of powers, because I could pair any power with a value of 64 with any power with a 0 exponent (because a power with a 0 exponent is equal to 1).

21. a) $1 \text{ km} = 1000 \text{ m}$
$= 10^3 \text{ m}$
$1 \text{ m} = 100 \text{ cm}$, so
$10^3 \text{ m} = 10^3 \times 100 \text{ cm}$
$= 10^3 \times 10^2 \text{ cm}$
$= 10^5 \text{ cm}$
There are 10^5 cm, or 100 000 cm in 1 km.

b) $1 \text{ km} = 10^5 \text{ cm}$
$1 \text{ cm} = 10 \text{ mm}$, so
$10^5 \text{ cm} = 10^5 \times 10^1 \text{ mm}$
$= 10^6 \text{ mm}$
There are 10^6 mm, or 1 000 000 mm in 1 km.

c) $1 \text{ km} = 1000 \text{ m}$
$= 10^3 \text{ m}$
$\frac{1}{10^3} \text{ km} = \frac{10^3}{10^3} \text{ m}$
$= 1 \text{ m}$
$10^5 \left(\frac{1}{10^3}\right) \text{ km} = 10^5 \times 1 \text{ m}$
$= 10^5 \text{ m}$
$10^5 \text{ m} = \frac{10^5}{10^3} \text{ km}$
$= 10^2 \text{ km}$
There are 10^2 km, or 100 km in 10^5 m.

d) $1 \text{ m} = 1000 \text{ mm}$
$= 10^3 \text{ mm}$
$\frac{1}{10^3} \text{ m} = \frac{10^3}{10^3} \text{ mm}$
$= 1 \text{ mm}$
$10^9 \left(\frac{1}{10^3}\right) \text{ m} = 10^9 \times 1 \text{ mm}$
$= 10^9 \text{ mm}$
$10^9 \text{ mm} = \frac{10^9}{10^3} \text{ m}$
$= 10^6 \text{ m}$

There are 10^6 m, 1 000 000 m in 10^9 mm

22. a) $1\ km^2 = (1000\ m)^2$
$= 1000\ m \times 1000\ m$
$= 1\ 000\ 000\ m^2$
$= 10^6\ m^2$
$10^2\ km^2 = 10^2 \times 10^6\ m^2$
$= 10^8\ m^2$
There are $10^8\ m^2$, or 100 000 000 m^2 in $10^2\ km^2$.

b) $1\ m^2 = (100\ cm)^2$
$= 10\ 000\ cm^2$
$= 10^4\ cm^2$
$\frac{1}{10^4}\ m^2 = \frac{10^4}{10^4}\ cm^2$
$= 1\ cm^2$
$10^6(\frac{1}{10^4})\ m^2 = 10^6 \times 1\ cm^2$
$10^6\ cm^2 = \frac{10^6}{10^4}\ m^2$
$= 10^2\ m^2$
There are $10^2\ m^2$, or 100 m^2 in $10^6\ cm^2$.

c) $1\ cm^2 = (10\ mm)^2$
$= 100\ mm^2$
$= 10^2\ mm^2$
$10^6\ cm^2 = 10^6 \times 10^2\ mm^2$
$= 10^8\ mm^2$
There are $10^8\ mm^2$, or 100 000 000 mm^2 in $10^6\ cm^2$.

d) $1\ km^2 = (100\ 000\ cm)^2$
$= 10\ 000\ 000\ 000\ cm^2$
$= 10^{10}\ cm^2$
There are $10^{10}\ cm^2$, or 10 000 000 000 cm^2 in 1 km^2.